Core Mathematics C4 For Edexcel Advanced Level

Paper A
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. $$f(x) = \frac{3x}{(x-1)(x+2)}.$$

(*a*) Express $f(x)$ in partial fractions. **(3)**

(*b*) Evaluate $\int_2^3 f(x)dx$. **(4)**

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2. A curve has parametric equations

$$x = 1 - t^3,\ y = 1 + t^2.$$

(*a*) Find the value of the parameter t at the point (2, 2). **(1)**

(*b*) Find the equation of the tangent to the curve at (2, 2). **(4)**

3.

Figure 1

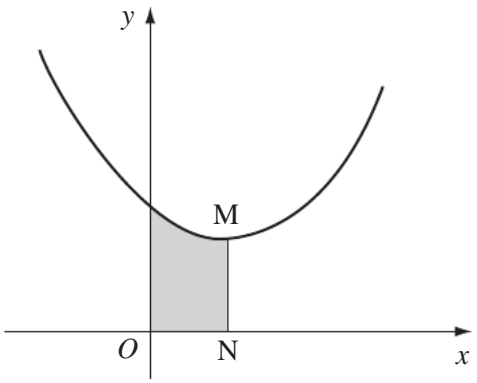

Figure 1 shows the curve with equation

$$y = e^x - 3x.$$

The minimum point on the curve is M and the line MN is parallel to the y-axis.

(*a*) Find the x-coordinate of M. **(2)**

(*b*) Show that the area of the shaded region can be written as

$$a - b(\ln 3)^2,$$

where the constants a and b are to be determined. **(5)**

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4. A curve has equation

$$4x^2 + 3y^2 - 2xy = 32.$$

(*a*) Find an expression for $\frac{dy}{dx}$ in terms of x and y, simplifying your answer.

(6)

(*b*) Find the gradient of the curve at the point (2, 4) and hence find the equation of a tangent to the curve at that point.

(3)

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5. In the series expansion of $(1 + kx)^n$, the coefficients of x and x^2 are -6 and 27 respectively. Find

(*a*) the value of k and the value of n,

(4)

(*b*) the coefficient of x^3 in the expansion,

(3)

(*c*) the set of values of x for which the expansion is valid.

(1)

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6. (*a*) Solve the differential equation

$$x^2\frac{dy}{dx} = y^2(4x^5 - 1),$$

given that $y = \frac{1}{2}$ when $x = 1$.

(6)

(*b*) Use the substitution $t = 1 + x^2$ to show that

$$\int_0^2 \frac{x^3}{(1+x^2)^{\frac{1}{2}}}\,dx = \frac{2}{3}(1 + \sqrt{5}).$$

(7)

7.

Figure 2

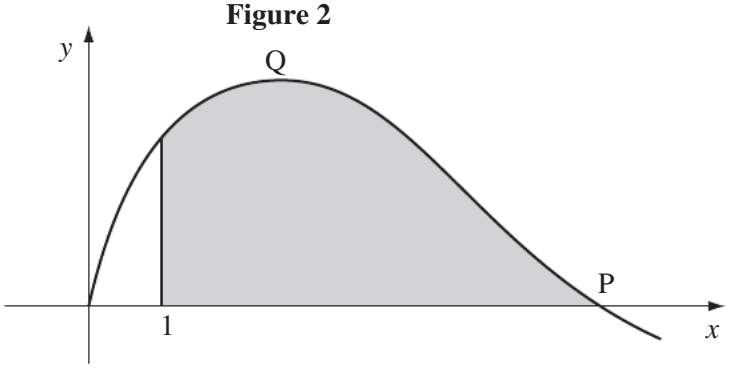

Figure 2 shows the graph of $y = 2x - x \ln x$. The graph crosses the x-axis at the point P and has a turning point at Q.

(a) Find the coordinates of Q.

Verify that $\dfrac{d^2y}{dx^2} < 0$ at this point. **(4)**

(b) Show that the coordinates of P are $(e^2, 0)$. **(2)**

(c) (i) Show that $\displaystyle\int_1^{e^2} x \ln x \, dx = \frac{3e^4 + 1}{4}$. **(6)**

(ii) Find the area of the shaded region bounded by the curve, the x-axis and the line $x = 1$. **(3)**

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8. The position vectors of A, B, C and D with respect to the origin are:

$$\underset{A}{\begin{pmatrix} 6 \\ 2 \\ 0 \end{pmatrix}} \quad \underset{B}{\begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}} \quad \underset{C}{\begin{pmatrix} 9 \\ 3 \\ 0 \end{pmatrix}} \quad \underset{D}{\begin{pmatrix} 4 \\ 8 \\ 2 \end{pmatrix}}$$

(a) The line through B and C is denoted by l_1 and the line through A and D is denoted by l_2. Show that l_1 has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -1 \\ -1 \end{pmatrix}$$ **(2)**

(b) Find an equation for l_2. **(2)**

(c) Find the position vector of the point where l_1 and l_2 intersect. **(4)**

(d) Calculate the acute angle between l_1 and l_2, correct to one decimal place. **(3)**

END **TOTAL 75 MARKS**